

BARTON COLLEGE
PRACTICE PLACEMENT TEST SOLUTIONS

1. Simplify: $20 + 4^3 \div (-8) = 20 + 64 \div (-8) = 20 + (-8) = 12$

(Order of operations from left to right: Parenthesis, Exponents, Multiplication, Division, Addition Subtraction)

2. Simplify: $(2a - 4) + 2(a - 5) - 3(a + 1) = 2a - 4 + 2a - 10 - 3a - 3 = a - 17$

3. Evaluate the expression: $4a^2 - 4ab + b^2$, when $a = 2$ and $b = 5$

$$4a^2 - 4ab + b^2 = 4(2)^2 - 4(2)(5) + (5)^2 = 16 - 40 + 25 = 1$$

4. Firefighters use the formula $S = 0.5P + 26$ to compute the horizontal range S in feet of water from a particular hose, where P is the nozzle pressure in pounds. Find the horizontal range if pressure is 90 lb.

Given $P = 90$ lb. Hence horizontal range $S = 0.5(90) + 26 = 71$ feet.

5. Simplify: $2x^2(-3x^2)^3 = 2x^2(-3)^3(x^2)^3 = 2x^2(-27)(x^6) = -54x^8$

6. Simplify: $\left(\frac{2u^{-5}v^2}{8w}\right)^{-2} = \left(\frac{8w}{2u^{-5}v^2}\right)^2 = \frac{(8)^2(w)^2}{(2)^2(u^{-5})^2(v^2)^2} = \frac{64w^2}{4u^{-10}v^4} = \frac{16w^2u^{10}}{v^4}$

7. Express in scientific notation: $0.0000056 = 5.6 \times 10^{-6}$ (count number of places from first non-zero digit to the decimal)

8. Expand: $1.20 \times 10^5 = 1.2 \times 10000 = 120000$

9. Solve: $\frac{1}{4}x - \frac{5}{8} = \frac{3}{8}$ Multiply both sides of the equation by 8 $\rightarrow 8\left(\frac{1}{4}x - \frac{5}{8} = \frac{3}{8}\right) \rightarrow 2x - 5 = 3 \rightarrow x = 4$

10. Solve: $8(x - 2) - 5(x + 4) = 20 + x$
 $8x - 16 - 5x - 20 = 20 + x$
 $3x - 36 = 20 + x$
 $3x - x = 20 + 36$
 $2x = 56 \rightarrow x = 28$

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11. Solve for m : $F = \frac{mv^2}{r} \rightarrow Fr = mv^2 \rightarrow \frac{Fr}{v^2} = m$.

12. Solve P: $A = P + Prt \rightarrow A = P(1 + rt) \rightarrow \frac{A}{1 + rt} = P$

13. Solve: $\frac{6}{x-5} = \frac{4}{x} \rightarrow 6x = 4(x-5) \rightarrow 6x = 4x - 20 \rightarrow 6x - 4x = -20 \rightarrow 2x = -20 \rightarrow x = -10$

14. Solve: $2|x-3| = 5 \rightarrow |x-3| = \frac{5}{2} \rightarrow x-3 = \frac{5}{2}$ or $x-3 = -\frac{5}{2}$
 $\rightarrow x = \frac{5}{2} + 3$ or $x = -\frac{5}{2} + 3 \rightarrow x = \frac{11}{2}$ or $x = \frac{1}{2}$

15. Solve: $3 - \frac{x}{x-4} = \frac{4}{4-x} \rightarrow (x-4)3 - (x-4)\left(\frac{x}{x-4}\right) = (x-4)\left(\frac{4}{4-x}\right)$
 $\rightarrow (x-4)3 - x = -(4-x)\left(\frac{4}{4-x}\right)$
 $\rightarrow 3x - 12 - x = -1(4) \rightarrow 2x - 12 = -4 \rightarrow 2x = 8 \rightarrow x = 4$

However, if $x = 4$ the denominator becomes zero in the original equation. **ANSWER: No Solution**

16. Simplify: $\frac{x^3 + x^2y - 6xy^2}{x^2 - 2xy} = \frac{x(x^2 + xy - 6y^2)}{x(x-2y)} = \frac{(x+3y)(x-2y)}{(x-2y)} = x+3y$

17. Simplify: $\frac{4x^2 - 1}{2x^2 + 5x - 3} = \frac{(2x+1)(2x-1)}{(2x-1)(x+3)} = \frac{2x+1}{x+3}$

18. Solve: $-3(2x - 3) \leq 27 \rightarrow -6x + 9 \leq 27 \rightarrow -6x \leq 18 \rightarrow x \geq -3$

19. Solve: $\frac{2}{3} + \frac{x}{5} < \frac{4}{15} \rightarrow 15\left(\frac{2}{3}\right) + 15\left(\frac{x}{5}\right) < 15\left(\frac{4}{15}\right) \rightarrow 10 + 3x < 4 \rightarrow 3x < -6 \rightarrow x < -2$

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20. John averaged 82 out of 100 on his first three tests. What was John's score on the fourth test if his average after the fourth test dropped to 79 out of 100?

$$\begin{aligned}\text{Test Average} &= \frac{T1+T2+T3+T4}{4} \rightarrow 79 = \frac{82+82+82+x}{4} \\ &\rightarrow (79)(4) = 246+x \rightarrow 316 = 246+x \rightarrow 70 = x\end{aligned}$$

21. The sales tax rate in Wilson County is 6.75%. Suppose total price of an item that you bought in Wilson County including taxes is \$14.93, what is the price (rounded to two decimal places) before tax?

$$\text{Price before tax} + 6.75\% \text{ sales tax} = \text{Total price} \rightarrow x + 0.0675x = 14.93 \rightarrow 1.0675x = 14.93 \rightarrow x = \$13.99$$

22. The long term parking rate at Raleigh-Durham Airport is \$2 per hour (or part of an hour) with \$10 daily maximum (12:00 a.m. to 12:00 a.m.). Suppose you park your car on Friday afternoon at 8:30 p.m. and pick it up on the following Tuesday at 9:30 a.m., what will be your parking fee?

$$\begin{aligned}\text{Friday} &= 3.5 \text{ hours} \rightarrow \$8 & \text{Parking Fee} &= 8 + 10 + 10 + 10 + 10 = \$48 \\ \text{Saturday} &= 24 \text{ hours} \rightarrow \$10 \\ \text{Sunday} &= 24 \text{ hours} \rightarrow \$10 \\ \text{Monday} &= 24 \text{ hours} \rightarrow \$10 \\ \text{Tuesday} &= 9.5 \text{ hours} \rightarrow \$10\end{aligned}$$

23. Solve: $2x(10x + 8) = -3(x+1) \rightarrow 20x^2 + 16x = -3x - 3 \rightarrow 20x^2 + 19x + 3 = 0$

$$\begin{aligned}\rightarrow x &= \frac{-19 \pm \sqrt{(19)^2 - 4(20)(3)}}{2(20)} = \frac{-19 \pm \sqrt{121}}{40} = \frac{-19 \pm 11}{40} \\ \rightarrow x &= \frac{-19+11}{40} \text{ or } \frac{-19-11}{40} = -\frac{1}{5} \text{ or } -\frac{3}{4}\end{aligned}$$

24. Solve: $(2x-3)^2 - 8 = 0 \rightarrow (2x-3)(2x-3) - 8 = 0 \rightarrow 4x^2 - 12x + 9 - 8 = 0 \rightarrow 4x^2 - 12x + 1 = 0$

$$\rightarrow x = \frac{-(-12) \pm \sqrt{(-12)^2 - 4(4)(1)}}{2(4)} = \frac{12 \pm \sqrt{144-16}}{8} = \frac{12 \pm \sqrt{128}}{8} = \frac{12 \pm 8\sqrt{2}}{8} = \frac{3 \pm 2\sqrt{2}}{2}$$

25. The profit, P, realized by a company varies directly as the number of products s it sells. If a company makes a profit of \$7800 on the sale of 325 products, what is the profit when the company sells 5000 products?

$$P = k s \rightarrow 7800 = k(325) \rightarrow \frac{7800}{325} = k \rightarrow 24 = k \rightarrow P = 24 s = 24(5000) = \$120,000$$

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26. If the voltage, V , in an electric circuit is held constant, the current I , is inversely proportional to the resistance, R . If current is 120mA (milliampere) when resistance is 5 ohms, find the current when the resistance is 15 ohms.

$$I = \frac{k}{R} \rightarrow 120 = \frac{k}{5} \rightarrow 600 = k \rightarrow I = \frac{600}{R} = \frac{600}{15} = 40 \text{ mA}$$

27. A 36 foot long tube is cut into two pieces with ratio 4:5. Find the length of the shorter piece.
The ratio 4:5 $\rightarrow 4x + 5x = 36 \rightarrow 9x = 36 \rightarrow x = 4 \rightarrow$ Shorter piece = $4(4) = 16$ feet

28. A large square pizza has 49 pieces (square slices). John, Jack and Jane ate all the pieces in the ratio 4:2:1 respectively. How many pieces did Jack eat?

The ratio 4:2:1 $\rightarrow 4x + 2x + x = 49 \rightarrow 7x = 49 \rightarrow x = 7 \rightarrow$ Jack ate $2(7) = 14$ pieces

29. Solve: $\sqrt{1-2x} + 1 = 3 \rightarrow \sqrt{1-2x} = 3 - 1 \rightarrow \sqrt{1-2x} = 2 \rightarrow (\sqrt{1-2x})^2 = 2^2 \rightarrow 1 - 2x = 4$
 $\rightarrow -2x = 4 - 1 \rightarrow -2x = 3 \rightarrow x = -\frac{3}{2}$

30. Solve for V given $r = \sqrt{\frac{V}{\pi h}} \rightarrow (r)^2 = \left(\sqrt{\frac{V}{\pi h}}\right)^2 \rightarrow r^2 = \frac{V}{\pi h} \rightarrow r^2 \pi h = V \rightarrow V = \pi h r^2$

31. Find the equation of the straight line passing through the points (2,-4) and (1,0).

$$y = mx + b \rightarrow \text{slope } m = \frac{0 - (-4)}{1 - 2} = \frac{4}{-1} = -4 \rightarrow y = -4x + b \rightarrow 0 = -4(1) + b \rightarrow 4 = b$$

$\rightarrow y = -4x + 4$

32. Determine the x and y intercepts of the graph of $7x - 5y = 35$

x-intercept: let $y = 0 \rightarrow 7x - 5(0) = 35 \rightarrow 7x = 35 \rightarrow x = 5$

y-intercept: let $x = 0 \rightarrow 7(0) - 5y = 35 \rightarrow -5y = 35 \rightarrow y = -7$

Answer: $(5, 0)$ and $(0, -7)$

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33. The linear relationship between the Fahrenheit scale and Centigrade scale for temperatures is given by $F = \frac{9}{5}C + 32$. Which of the following statements, if any, are **TRUE**?

A. If $C = 20^\circ$, then $F = \frac{9}{5}C + 32 = \frac{9}{5}(20) + 32 = 9(4) + 32 = 68$ TRUE

B. If $C = 40^\circ$, then $F = \frac{9}{5}C + 32 = \frac{9}{5}(40) + 32 = 9(8) + 32 = 104$ FALSE

ANSWER: **Only A**

34. John (J) is 5 years older than his sister Mary (M) who is 2 years younger than her brother Paul (P). If J, M and P denote their ages, which one of the following represents the given information?

John is 5 years older than Mary $\rightarrow J = M + 5$

Mary is 2 years younger than Paul $\rightarrow M = P - 2$

35. Solve the system: $\begin{cases} 3x - 5y = -4 \\ 3x - y = 4 \end{cases}$ Subtracting second equation from first gives $-5y - (-y) = -4 - 4$

$\rightarrow -5y + y = -8 \rightarrow -4y = -8 \rightarrow y = 2$.

The first equation becomes $3x - 5(2) = -4 \rightarrow 3x - 10 = -4 \rightarrow 3x = 6 \rightarrow x = 2$

Answer: **(2, 2)**

36. The sum of two numbers is 31. Twice the smaller number is 11 more than the larger number. The positive difference between the numbers is

Let x be the larger number and y the smaller.

The sum of two numbers is 31.

$\rightarrow x + y = 31 \rightarrow y = 31 - x$

Twice the smaller number is 11 more than the larger number.

$\rightarrow 2y = x + 11$

$\rightarrow 2(31 - x) = x + 11$

$\rightarrow 62 - 2x = x + 11 \rightarrow 62 - 11 = x + 2x \rightarrow 51 = 3x \rightarrow 17 = x$

$x + y = 31 \rightarrow 17 + y = 31 \rightarrow y = 31 - 17 \rightarrow y = 14$

The positive difference between the number is $x - y = 17 - 14 = 3$

37. Find the coordinates of a point A whose distance from the origin (0, 0) is 5 units.

Distance of a point A (x, y) from the origin (0, 0) = $\sqrt{(x-0)^2 + (y-0)^2} = \sqrt{x^2 + y^2}$

Distance of A (3, 3) from the origin = $\sqrt{3^2 + 3^2} = \sqrt{18} \neq 5$

Distance of A (-3, 2) from the origin = $\sqrt{(-3)^2 + 2^2} = \sqrt{13} \neq 5$

Distance of A (4, -3) from the origin = $\sqrt{4^2 + (-3)^2} = \sqrt{25} = 5$

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38. Consider the circle given by the equation $(x - 2)^2 + (y + 1)^2 = 5$. Find the center and radius.

The equation of a circle with center (h, k) and radius r is $(x - h)^2 + (y - k)^2 = r^2$

$(x - 2)^2 + (y + 1)^2 = 5 \rightarrow (x - 2)^2 + (y - (-1))^2 = (\sqrt{5})^2 \rightarrow$ Center is $(2, -1)$ and radius is $\sqrt{5}$

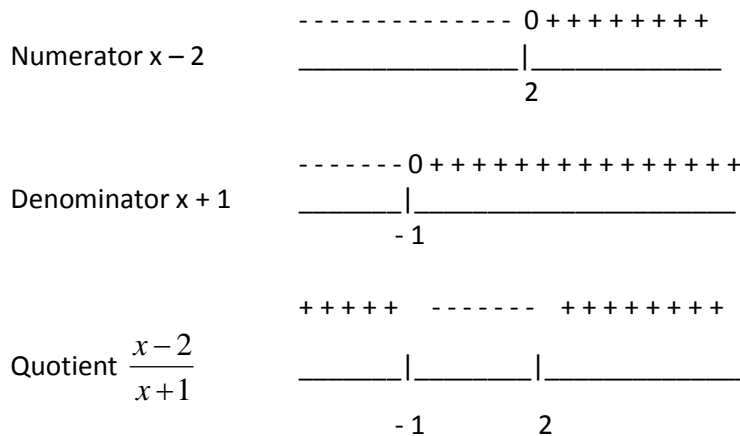
39. The inequality $|8 - x| < 8$ is equivalent to

By definition of absolute value $|8 - x| < 8 \rightarrow -8 < 8 - x < 8$
 Subtract 8 from the three sides $-8 - 8 < 8 - x - 8 < 8 - 8 \rightarrow -16 < -x < 0$
 Divide three sides by -1 and this will reverse the inequalities $16 > x > 0$
 Rewriting with zero on the left $\rightarrow 0 < x < 16$

40. The inequality $|x + 4| \geq 1$ is equivalent to

By definition of absolute value $|x + 4| \geq 1 \rightarrow x + 4 \leq -1$ or $x + 4 \geq 1 \rightarrow x \leq -1 - 4$ or $x \geq 1 - 4$
 $\rightarrow x \leq -5$ or $x \geq -3$

41. The interval solution to the inequality $\frac{x - 2}{x + 1} > 0$ is



Hence $\frac{x - 2}{x + 1} > 0$ when x is to the left of -1 or right of $2 \rightarrow$ Answer: $(-\infty, -1) \cup (2, +\infty)$

42. Let $f(x) = \sqrt{1 - 2x}$. Find $f(a - 1)$

$f(a - 1) = \sqrt{1 - 2(a - 1)} = \sqrt{1 - 2a + 2} = \sqrt{3 - 2a}$

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43. Let $f(x) = 2 - x^2$ and $g(x) = 2x - 1$. Which of the following, if any, is **false**?

a) $(f + g)(0) = f(0) + g(0) = [2 - 0^2] + [2(0) - 1] = [2] + [-1] \neq -2$. Hence it is **false**.

44. Let $f(x) = 2 - x^2$ and $g(x) = 2x - 1$. Which of the following, if any, is **true**?

a) $(f \circ g)(0) = f[g(0)] = f[2(0) - 1] = f(-1) = 2 - (-1)^2 = 2 - 1 = 1 \neq -2$

b) $(g \circ f)(0) = g[f(0)] = g[2 - 0^2] = g[2] = 2(2) - 1 = 3$ **TRUE**

c) $(f \circ f)(x) = f[f(x)] = f[2 - x^2] = 2 - (2 - x^2)^2 = 2 - (4 - 4x^2 + x^4) \neq 4 - 4x^2 + x^4$

d) $(g \circ g)(x) = g[g(x)] = g[2x - 1] = 2(2x - 1) - 1 = 4x - 2 - 1 \neq 4x^2 - 4x + 1$

45. Let $f(x) = 3 - 2x$. Find the difference quotient $\frac{f(x+h) - f(x)}{h}$

$$\frac{f(x+h) - f(x)}{h} = \frac{[3 - 2(x+h)] - [3 - 2x]}{h} = \frac{[3 - 2x - 2h] - [3 - 2x]}{h} = \frac{-2h}{h} = -2$$

46. Consider the quadratic function $f(x) = 2x^2 - 4x + 1$. Find the vertex of the graph of $f(x)$.

Given a quadratic function $f(x) = ax^2 + bx + c$ the coordinates of the vertex are $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$

Here $a = 2$, $b = -4$ and $c = 1$. Hence $-\frac{b}{2a} = -\frac{-4}{2(2)} = 1$ and $f\left(-\frac{b}{2a}\right) = f(1) = 2(1)^2 - 4(1) + 1 = -1$

Answer: The vertex of the graph is at **(1, -1)**

47. The temperature, in degrees Fahrenheit, over a twelve hour period is given by the function $T(t) = -0.5t^2 + 6t + 30$, where $t = 0$ denotes 6:00 a.m. When is the morning temperature 47.5° F?

Solve: $47.5 = -0.5t^2 + 6t + 30 \rightarrow -0.5t^2 + 6t + 30 - 47.5 = 0 \rightarrow -0.5t^2 + 6t - 17.5 = 0$

Divide throughout by $-0.5 \rightarrow t^2 - 12t + 35 = 0 \rightarrow (t - 5)(t - 7) = 0 \rightarrow t = 5$ or $7 \rightarrow 11$ a.m. or 1 p.m.

Answer: **11 a.m.**

48. Simplify and express in the form $a + bi$: $(-2 + i)(3 + 2i) = -6 - 4i + 3i + 2i^2 = -6 - i + 2(-1) = -8 - i$

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49. Simplify and express in the form $a + bi$: $\frac{-4i}{1+i} = \frac{-4i(1-i)}{(1+i)(1-i)} = \frac{-4i+4i^2}{1-i+i-i^2} = \frac{-4i+4(-1)}{1-(-1)} = \frac{-4-4i}{2}$
 $= -2 - 2i$

50. Find the domain of the function $f(x) = \frac{x+3}{x^2+1}$.

Since the denominator $x^2 + 1 \neq 0$ for any real x , the domain is **all real numbers**.

51. Find the equation of horizontal asymptote of the function $f(x) = \frac{x+3}{x^2-1}$

Since the denominator has a higher degree than numerator, the horizontal asymptote is the x-axis. **$y = 0$**

52. Find the inverse function for $f(x) = \frac{x}{3} - 2$.

To find the inverse replace (x, y) by (y, x) and solve for y . Hence $y = \frac{x}{3} - 2$ becomes $x = \frac{y}{3} - 2$

Multiplying both sides by 3 $\rightarrow 3x = y - 6 \rightarrow 3x + 6 = y \rightarrow f^{-1}(x) = 3x + 6$

53. Which of the following pairs of exponential and logarithmic forms is **false**?

$(1/2)^{-2} = 4$; $\log_4(1/2) = -2$ is false since $(1/2)^{-2} = 4 \Rightarrow \log_{1/2}(4) = -2$

54. Write in terms of $\log(x)$, $\log(y)$, $\log(z)$:

$$\log\left(\frac{y\sqrt{z}}{x^2}\right) = \log(y\sqrt{z}) - \log(x^2) = \log(y) + \log(z^{0.5}) - 2\log(x) = \log(y) + 0.5\log(z) - 2\log(x)$$

55. Find the hydronium-ion concentration, H^+ , of a solution given $\text{pH} = 7.1$

$$\text{pH} = -\log(H^+) \rightarrow 7.1 = -\log(H^+) \rightarrow 7.1 = -\log_{10}(H^+) \rightarrow -7.1 = \log_{10}(H^+) \rightarrow H^+ = 10^{-7.1}$$

$$\rightarrow H^+ = 7.9 \times 10^{-8} \text{ moles per liter}$$

56. Solve: $e^{2x-1} = 32 \rightarrow 2x-1 = \log_e(32) \rightarrow 2x = \ln(32) + 1 \rightarrow x = \frac{1 + \ln(32)}{2}$

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57. Solve the equation: $\log_2(x+1) - \log_2(x-1) = 2 \Rightarrow \log_2\left(\frac{x+1}{x-1}\right) = 2 \Rightarrow \frac{x+1}{x-1} = 2^2$

$\rightarrow x+1 = 4(x-1) \rightarrow x+1 = 4x-4 \rightarrow 1+4 = 4x-x \rightarrow 5 = 3x \rightarrow x = \frac{5}{3}$.

58. Suppose the population of a town is given by the model $P(t) = 17250\left(\frac{1}{2}\right)^{t/10}$, where t denotes the number years since 2000. Which of the following statements is **true**?

- a) The population in 2000 was 8625. **False** since when $t = 0$, $P(0) = 17250$.
b) The population doubles every 10 years. **False** since when $t = 10$, $P(10) = 8625$.
c) **The population is halved every ten years. TRUE** since when $t = 10$, $P(10) = 8625 = \frac{1}{2}(17250)$.
d) The population is growing by 50% every ten years. **FALSE** since population is decreasing.

59. If the angle $\theta = \frac{3\pi}{5}$ radians, then **since $\pi = 180^\circ$** , $\theta = \frac{3\pi}{5} = \frac{3(180)}{5} = 108^\circ$

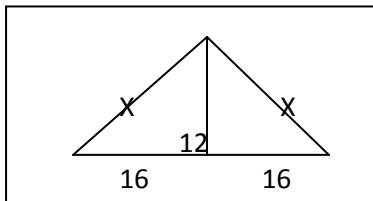
Answer: $90^\circ < \theta < 180^\circ$

60. If the light beam makes one complete revolution every 20 seconds, how long will it take to sweep and angle of 150° ?

1 revolution = $360^\circ \rightarrow$ in 1 second the light beam sweeps $\frac{360}{20} = 18^\circ \rightarrow$ to sweep $150^\circ = \frac{150}{18} = 8.3$ sec

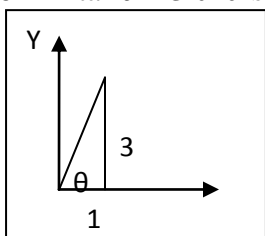
Answer: **between 7 and 10 seconds**

61. Given an isosceles triangle with base length 32 cm and altitude 12 cm, find the length of the congruent sides.



$X^2 = 16^2 + 12^2 = 400 \rightarrow X = 20$ cm

62. If $\tan \theta = 3$ and $\sin \theta > 0$, then $\cos \theta$ equals



Since $\tan \theta = 3 > 0$ and $\sin \theta > 0$, the angle θ is in the Quadrant I

$\tan \theta = \frac{\text{opp}}{\text{adj}} = 3 = \frac{3}{1} \rightarrow \text{hyp} = \sqrt{1^2 + 3^2} = \sqrt{10} \rightarrow \cos \theta = \frac{\text{adj}}{\text{hyp}} = \frac{1}{\sqrt{10}}$

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63. If $\csc \theta = \frac{13}{5}$ and $\cos \theta < 0$, then $\cot \theta$ equals

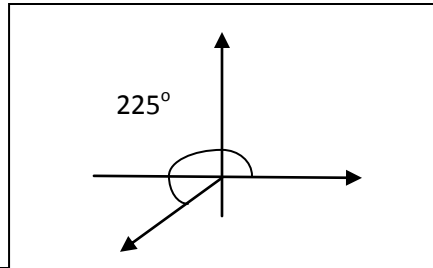
Since $\csc \theta = \frac{13}{5} > 0 > 0$ and $\cos \theta < 0$, the angle θ is in the Quadrant II $\rightarrow \cot \theta < 0$

$$\csc \theta = \frac{\text{hyp}}{\text{opp}} = \frac{13}{5} \rightarrow \text{adj} = \sqrt{13^2 - 5^2} = \sqrt{144} = 12 \rightarrow \cot \theta = \frac{\text{adj}}{\text{opp}} = -\frac{12}{5}$$

64. Find the exact value of $\csc(225^\circ)$

225° is in Quadrant III $\rightarrow \csc(225^\circ) < 0$
The reference angle is 45° .

$$\rightarrow \csc(225^\circ) = -\csc(45^\circ) = -\sqrt{2}$$



65. Find the exact value of $\cot(420^\circ)$. Since $420^\circ = 360^\circ + 60^\circ$

$$\cot(420^\circ) = \cot(60^\circ) = \frac{1}{\tan 60^\circ} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

66. If the angle θ in standard position meets the unit circle at $\left(\sqrt{\frac{5}{6}}, -\sqrt{\frac{1}{6}}\right)$, find the value of the functions $\sin(\theta)$ and $\cos(\theta)$.

Since the point is in Quadrant IV, $\sin(\theta) < 0$ and $\cos(\theta) > 0$. Further on the unit circle $\sin(\theta) = \frac{y}{r}$ and

$$\cos(\theta) = \frac{x}{r} \text{ with } r = 1. \text{ Hence } \sin \theta = -\sqrt{\frac{1}{6}} \text{ and } \cos \theta = \sqrt{\frac{5}{6}}$$

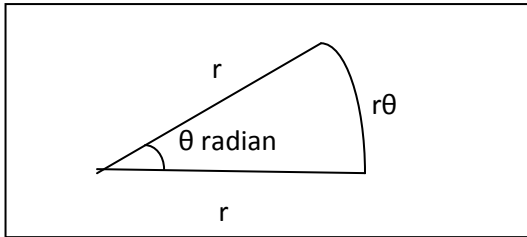
67. Find the expression that is equal to $\frac{1 + \sin \theta}{1 - \sin \theta} = \frac{1 + \sin \theta}{\frac{1 - \sin \theta}{\sin \theta}} = \frac{\frac{1}{\sin \theta} + 1}{\frac{1}{\sin \theta} - 1} = \frac{\csc \theta + 1}{\csc \theta - 1}$

68. Find the expression that is equal to

$$\begin{aligned} (\tan \theta + \cot \theta)^2 &= \tan^2 \theta + 2 \tan \theta \cot \theta + \cot^2 \theta = \tan^2 \theta + 2 + \cot^2 \theta = (\tan^2 \theta + 1) + (\cot^2 \theta + 1) \\ &= \sec^2 \theta + \csc^2 \theta \end{aligned}$$

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69. The minute hand of a clock is 6cm long. How far does the tip of the minute hand travel in 15 minutes?



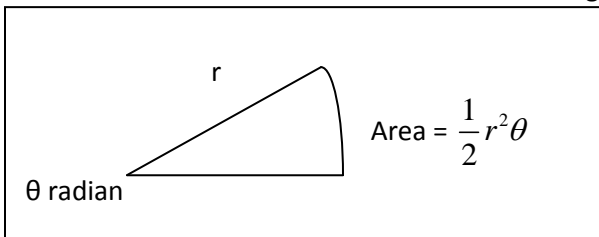
$$1 \text{ hr} = 60 \text{ minutes} = 360^\circ = 2\pi \text{ radian}$$

$$\rightarrow 15 \text{ minutes} = 90^\circ = \pi/2 \text{ radian}$$

$$\text{Distance travelled by tip of minute hand} = 6(\pi/2)$$

$$= 3\pi \text{ cm}$$

70. Find the area of a sector of a circle with central angle $\theta = 3$ radians, if the radius of the circle is 6 in.



$$\text{Area} = \frac{1}{2} r^2 \theta = \frac{1}{2} (6)^2 (3) = 54 \text{ in}^2.$$

71. The area of the sector of a circle with central angle of $\theta = 2$ radians is 16m^2 . Find the radius of the circle.

$$\text{Area} = \frac{1}{2} r^2 \theta \rightarrow 16 = \frac{1}{2} r^2 (2) \rightarrow 16 = r^2 \rightarrow r = 4 \text{ m}$$

72. Solve $4\cos\theta + 6 = 5(\cos\theta + 1)$, $0 \leq \theta < 360^\circ$

$$4\cos\theta + 6 = 5(\cos\theta + 1) \rightarrow 4\cos\theta + 6 = 5\cos\theta + 5 \rightarrow 1 = \cos\theta \rightarrow \theta = 0^\circ$$

73. Solve $(2\sin\theta - 3)(\cos\theta + 2) = 0$, $0 \leq \theta \leq \pi$

$$(2\sin\theta - 3)(\cos\theta + 2) = 0 \rightarrow 2\sin\theta - 3 = 0 \text{ or } \cos\theta + 2 = 0 \rightarrow \sin\theta = \frac{3}{2} \text{ or } \cos\theta = -2$$

Since the range of both functions $f(\theta) = \sin\theta$ and $f(\theta) = \cos\theta$ is $[-1, 1]$ this equation cannot be solved.

Answer: **No solution.**

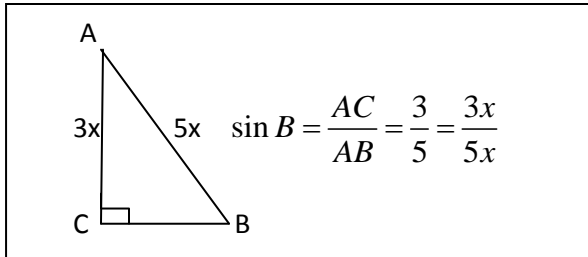
74. Solve $2\sin^2\theta = \sin\theta + 1$, $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$

$$2\sin^2\theta = \sin\theta + 1 \rightarrow 2\sin^2\theta - \sin\theta - 1 = 0 \rightarrow (2\sin\theta + 1)(\sin\theta - 1) = 0 \rightarrow \sin\theta = -\frac{1}{2} \text{ or } \sin\theta = 1$$

$$\rightarrow \theta = -\frac{\pi}{6}, \frac{\pi}{2}$$

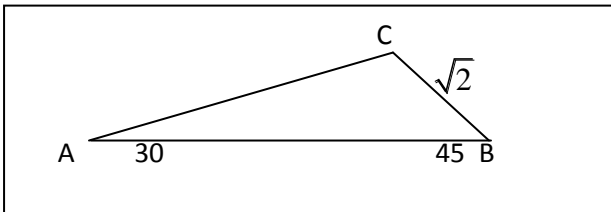
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75. In a right triangle ABC with $m\angle C = 90^\circ$, if $AC = 12$ and $\sin(B) = \frac{3}{5}$, find AB.



If $AC = 12 \rightarrow 3x = 12 \rightarrow x = 4 \rightarrow AB = 5x = 20$

76. In triangle ABC with $m\angle A = 30^\circ$, $m\angle B = 45^\circ$ and $BC = \sqrt{2}$, find AC.

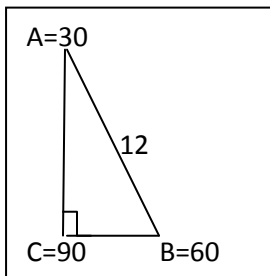


Using the Sine Formula $\frac{AC}{\sin B} = \frac{BC}{\sin A}$

$\rightarrow \frac{AC}{\sin 45} = \frac{\sqrt{2}}{\sin 30} \rightarrow \frac{AC}{1/\sqrt{2}} = \frac{\sqrt{2}}{1/2}$

$\rightarrow \sqrt{2}AC = 2\sqrt{2} \rightarrow AC = 2$

77. In triangle ABC with $m\angle A = 30^\circ$, $m\angle B = 60^\circ$ and $AB = 12$, find BC.



$m\angle A = 30^\circ$, $m\angle B = 60^\circ \rightarrow m\angle C = 180 - 30 - 60 = 90^\circ$

In right triangle ABC, $\cos(B) = \frac{BC}{AB} \rightarrow \cos(60) = \frac{BC}{12}$

$\rightarrow BC = 12 \cos(60) = 12 \left(\frac{1}{2}\right) = 6$

78. Eliminate the parameter t in the given parametric equation $\begin{cases} x = \cos(t) \\ y = 3 + \sin(t) \end{cases} \Rightarrow \begin{cases} x = \cos(t) \\ y - 3 = \sin(t) \end{cases}$

$\rightarrow x^2 + (y - 3)^2 = \cos^2 t + \sin^2 t \rightarrow x^2 + (y - 3)^2 = 1$

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79. Eliminate the parameter t in the given parametric equation $\begin{cases} x = \sin(t)\cos(t) \\ y = \sin(2t) \end{cases} \Rightarrow \begin{cases} x = \sin(t)\cos(t) \\ y = 2\sin(t)\cos(t) \end{cases}$

$\rightarrow y = 2x$

80. Given vectors $\vec{u} = \langle -4, -3 \rangle$ and $\vec{v} = \langle 2, 1 \rangle$, which of the following statements, if any, is **false**.

a) $|\vec{u}| = \sqrt{(-4)^2 + (-3)^2} = \sqrt{25} = 5$ TRUE

b) $\vec{u} + 2\vec{v} = \langle -4, -3 \rangle + 2\langle 2, 1 \rangle = \langle -4 + 2(2), -3 + 2(1) \rangle = \langle 0, -1 \rangle \neq \vec{0}$ **False**

c) $\vec{v} - \vec{u} = \langle 2, 1 \rangle - \langle -4, -3 \rangle = \langle 2 - (-4), 1 - (-3) \rangle = \langle 6, 4 \rangle$ TRUE

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ANSWER KEY

1.	c	21.	b	41.	c	61.	c
2.	b	22.	c	42.	c	62.	c
3.	b	23.	c	43.	a	63.	b
4.	d	24.	a	44.	b	64.	a
5.	d	25.	a	45.	d	65.	b
6.	c	26.	a	46.	c	66.	b
7.	a	27.	b	47.	b	67.	b
8.	c	28.	c	48.	c	68.	d
9.	a	29.	b	49.	b	69.	d
10.	b	30.	c	50.	a	70.	a
11.	d	31.	a	51.	d	71.	c
12.	c	32.	a	52.	d	72.	a
13.	c	33.	a	53.	b	73.	a
14.	b	34.	d	54.	b	74.	a
15.	d	35.	b	55.	a	75.	a
16.	d	36.	b	56.	a	76.	b
17.	d	37.	c	57.	c	77.	b
18.	c	38.	c	58.	c	78.	d
19.	c	39.	d	59.	b	79.	b
20.	d	40.	b	60.	d	80.	b.